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Inhomogeneous Dependency Modelling with Time Varying Copulae

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Abstract

Measuring dependence in a multivariate time series is tantamount to modelling its dynamic structure in space and time. In the context of a multivariate normally distributed time series, the evolution of the covariance (or correlation) matrix over time describes this dynamic. A wide variety of applications, though, requires a modelling framework different from the multivariate normal. In risk management the non-normal behaviour of most financial time series calls for nonlinear (i.e. non-gaussian) dependency. The correct modelling of non-gaussian dependencies is therefore a key issue in the analysis of multivariate time series. In this paper we use copulae functions with *adaptively estimated* time varying parameters for modelling the distribution of returns, free from the usual normality assumptions. Further, we apply copulae to estimation of *Value-at-Risk* (VaR) of a portfolio and show its better performance over the *RiskMetrics* approach, a widely used methodology for VaR estimation.

JEL classification: C 14

Keywords: Value-at-Risk, time varying copula, adaptive estimation, nonparametric estimation

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1 Introduction

Time series of financial data are high dimensional and have typically a non-gaussian behavior. The classical linear modelling therefore fails to reproduce the stylized facts (i.e. fat tails, asymmetry), Granger (2003). A correct understanding of the time varying multivariate (conditional) distribution is vital to many standard applications in finance: portfolio selection, option pricing, asset pricing models, Value-at-Risk (VaR) etc.

The dependency (over time) of asset returns is especially important in risk management since the *profit and loss (P&L) function* determines the Value-at-Risk. More precisely, Value-at-Risk of a portfolio is determined by the multivariate distribution of risk factor increments. If $w = (w_1, \ldots, w_d)^\top \in \mathbb{R}^d$ denotes a portfolio of positions on d assets and $S_t = (S_{1,t}, \ldots, S_{d,t})^\top$ a non-negative random vector representing the prices of the assets at time t, the value V_t of the portfolio w is given by

$$V_t = \sum_{j=1}^{d} w_j S_{j,t}.$$

The random variable

$$L_t = (V_t - V_{t-1}), \quad S_{j,0} = 0 \tag{1.1}$$

called *profit and loss (P&L) function*, expresses the change in the portfolio value between two subsequent time points. Defining the *log-returns* $X_t = \log S_t - \log S_{t-1}$, (1.1) can be written as

$$L_t = \sum_{j=1}^{d} w_j S_{j,t-1} \left\{ \exp(X_{j,t}) - 1 \right\}. \tag{1.2}$$

The distribution function of L_t is given by $F_{t,L_t}(x) = P_t(L_t \leq x)$. The *Value-at-Risk* at level α from a portfolio w is defined as the α-quantile from F_{t,L_t}:

$$VaR_t(\alpha) = F^{-1}_{t,L_t}(\alpha). \tag{1.3}$$

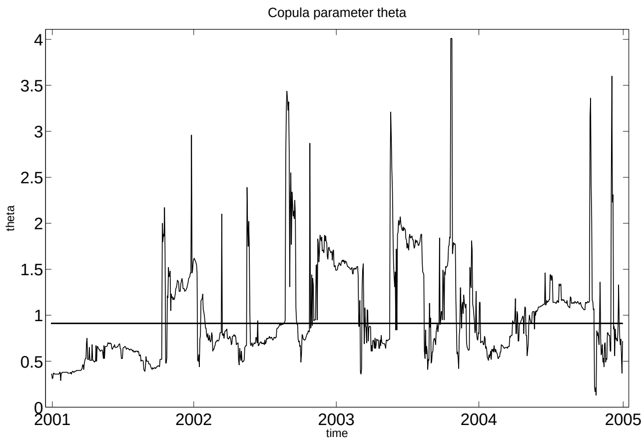


Figure 1: Dependence over time for DaimlerChrysler, Volkswagen, Bayer, BASF, Allianz and Münchener Rückversicherung, 20000101-20041231.

It follows from (1.2) and (1.3) that F_{t,L_t} depends on the specification of the d-dimensional distribution of the risk factors X_t. Thus, modelling their distribution over time is essential to obtain the quantiles (1.3).

The *RiskMetrics* technique, a widely used methodology for VaR estimation assumes that the log-returns follow a multivariate normal distribution. Here $\mathcal{L}(X_t) = N_d(0, \Sigma_t)$ a $d-$ dimensional multivariate distribution. A more general approach is based on copulae which avoids the procrustes bed of a normality assumptions resulting in better fits of the empirical characteristics (e.g. fat tails, tail dependency) of financial returns. Modelling the distribution of returns by copulae with time varying parameters, can therefore be expected to perform better. The question though is how to steer the time varying copulae parameters. This is exactly the focus of this paper.

Figure 1 shows the time varying copula parameter for DaimlerChrysler, Volkswagen, Bayer, BASF, Allianz and Münchener Rückversicherung from 1.Jan 2000 (20000101) to 31.Dec 2004 (20041231). In contrast the "global" copula parameter is shown by a constant horizontal line. The "local" choice of copula is performed via an adaptive estimation method based on Spokoiny (2007). The *adaptive estimation* is based on the assumption of local homogeneity: for every time point there exists an interval of time homogeneity in which the copula parameter can be well approximated by a constant. This interval is recovered from the data using local change point analysis. For a stock portfolio, we estimate copulae with time varying parameters and simulate the VaR accordingly. Backtesting underlines the improved performance of the proposed *adaptive time varying copulae fitting*.

This paper is organized as follows: section 2 presents the basic copulae definitions and introduces modelling log-returns with copulae. Section 3 discusses the VaR and its estimation procedure and section 4 describes three possible copulae estimation procedures. The adaptive estimation and the moving window approach are presented in section 5 and in applied on simulated data in section 6. Using real data, the performance of the copula-based VaR estimation in comparison with *RiskMetrics* approach is evaluated by means of *Backtesting* in section 7.

2 A short introduction into copulae

Copula functions have a long history in probability theory and statistics: they are well known and can be found in a variety of the financial literature. The word copula first appears in Sklar (1959), although the ideas related to copulae originate in Hoeffding (1940). Since that, copula funcions have been studied in a variety of the statistics literature such as Nelsen (1998), Mari and Kotz (2001) and Franke et al. (2004). The application of copulae in finance is very recent: the idea first appears in Embrechts et al. (1999) in connection with correlation as a measure of dependence. Futher financial applications can be found in Embrechts et al. (2003b) and Embrechts et al. (2003a). Copulae constitute an essential part in quantitative finance, see Härdle et al. (2002), and as mentioned above are recognized as an important tool in VaR calculations.

Copulae represent an elegant concept of connecting marginals with joint cummulative distribution functions. Copulae are functions that join or "couple" multivariate distribution functions to their 1-dimensional marginal distribution functions. They can preliminary be defined as multvariate distribution functions on the unit cube $[0,1]^d$ with uniform-(0,1) marginals. Copulae provide a natural way for measuring the dependence structure between random variables. The most reasonable way to define copulae regarding their applications is obtained by using Sklar's theorem:

Definition 2.1. *A d-dimensional copula is a function $C : [0,1]^d \to [0,1]$ with uniform-(0,1) marginals. If F is a d-dimensional distribution function with marginals $F_1 \ldots, F_d$, then there exists a copula C with*

$$F(x_1, \ldots, x_d) = C\{F_1(x_1), \ldots, F_d(x_d)\} \tag{2.1}$$

for every $x_1, \ldots, x_d \in \overline{\mathbb{R}}$. If $F_1, \ldots, F_d$ are continuous, then C is unique. Converserly, if C is a copula and $F_1, \ldots, F_d$ are distribution functions, then the function F defined in (2.1) is a joint distribution function with marginals $F_1, \ldots, F_d$.

Sklar's theorem reveals that the multivariate dependence structure and the univariate marginals can be modelled separately and that the dependence structure is modelled by means of copulae. For all $u = (u_1, \ldots, u_d)^\top \in [0,1]^d$, every copula C satisfies

$$W(u_1, \ldots, u_d) \leq C(u_1, \ldots, u_d) \leq M(u_1, \ldots, u_d) \text{ where}$$

$$\begin{aligned} M(u_1, \ldots, u_d) &= \min(u_1, \ldots, u_d) \text{ and} \\ W(u_1, \ldots, u_d) &= \max\left(\sum_{i=1}^{d} u_i - d + 1, 0\right). \end{aligned}$$

$M(u_1, \ldots, u_d)$ is called *Fréchet-Hoeffding upper bound* and $W(u_1, \ldots, u_d)$ the *Fréchet-Hoeffding lower bound*. They have been introduced in Fréchet (1951). For $d = 2$, the lower and the upper Fréchet-Hoeffding bounds are themselves copulae: they introduce the bivariate distribution functions of random vectors $(U, 1-U)^\top$ respectively $(U, U)^\top$, whereas U is the uniform-(0,1) random variable. In this case, the perfect negative dependence is described by W whereas M describes perfect positive dependence. For $d > 2$ W is a copula while M is not, see Nelsen (1998) or Embrechts et al. (1999).

If $X = (X_1, \ldots, X_d)^\top$ is a random vector with distribution $X \sim F_X$ and continuous marginals $X_j \sim F_{X_j}$, the copula of X is the distribution function C_X of $u = (u_1, \ldots, u_d)^\top$ where $u_j = F_{X_j}(x_j)$:

$$C_X(u_1, \ldots, u_d) = F_X\{F_{X_1}^{-1}(u_1), \ldots, F_{X_d}^{-1}(u_d)\}. \tag{2.2}$$

For an absolutely continuous copula C, the *copula density* is defined as

$$c(u_1, \ldots, u_d) = \frac{\partial^d C(u_1, \ldots, u_d)}{\partial u_1 \ldots \partial u_d}. \tag{2.3}$$

Some d-dimensional parametric copulae are presented below.

2.1 Gaussian copula for Gaussian marginals

The Gaussian copula represents the *dependence structure* of the multivariate normal distribution. For $Y = (Y_1, \ldots, Y_d)^\top \sim N_d(0, \Psi)$, Ψ a correlation matrix, the Gaussian copula is:

$$C_\Psi^{Ga}(u_1, \ldots, u_d) = F_Y\{\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_d)\} \tag{2.4}$$

$$= \int_{-\infty}^{\Phi^{-1}(u_1)} \ldots \int_{-\infty}^{\Phi^{-1}(u_d)} 2\pi^{-\frac{d}{2}} \mid \Psi \mid^{-\frac{1}{2}} \exp\left(-\frac{1}{2} r^\top \Psi^{-1} r\right) dr_1 \ldots dr_d.$$

Defining $\zeta_j = \Phi^{-1}(u_j)$, $\zeta = (\zeta_1, \ldots, \zeta_d)^\top$, the *density of the Gaussian copula* is

$$c_\Psi^{Ga}(u_1, \ldots, u_d) = \mid \Psi \mid^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\zeta^\top(\Psi^{-1} - \mathcal{I}_d)\zeta\right\}.$$

The copula parameter is here Ψ.

2.2 Gumbel copula

$$C_\theta(u_1, \ldots, u_d) = \exp\left[-\left\{\sum_{j=1}^{d}(-\log u_j)^\theta\right\}^{\theta^{-1}}\right], \quad 1 \le \theta \le \infty.$$

For $\theta > 1$ this copula presents upper tail dependence while for $\theta = 1$ it reduces to the product copula (independence):

$$C_\theta(u_1, \ldots, u_d) = \prod_{j=1}^{d} u_j.$$

When θ tends to infinity we obtain the Fréchet-Hoeffding upper bound:

$$C_\theta(u_1, \ldots, u_d) \overset{\theta\to\infty}{\longrightarrow} \min(u_1, , \ldots, u_d).$$

The copula parameter is θ and for $\theta \to \infty$ it indicates maximal dependence.

2.3 Clayton copula

$$C_\theta(u_1, \ldots, u_d) = \left\{\left(\sum_{j=1}^{d} u_j^{-\theta}\right) - d + 1\right\}^{-\theta^{-1}}, \quad \theta > 0$$

where the density of the Clayton copula is:

$$c_\theta(u_1, \ldots, u_d) = \prod_{j=1}^{d}\{1 + (j-1)\theta\}u_j^{-(\theta+1)}\left(\sum_{j=1}^{d} u_j^{-\theta} - d + 1\right)^{-(\theta^{-1}+d)}.$$

As the copula parameter θ tends to infinity, dependence becomes maximal and as θ tends to zero, we have independence. As θ goes to 1, copula achieves the lower Fréchet bound. The Clayton copula can mimic lower tail dependence but no upper tail dependence.

2.4 Kullback-Leibler Divergence and Copulae

For our further analysis of a jump in the copula parameter θ, the concept of Kullback-Leibler divergence will be required. Let X denote a random variable distributed as follows: $X \sim C_\theta\{F_{X_1}(x_1), \dots, F_{X_d}(x_d)\}$. The density function of X is given by

$$f_\theta(x_1, \dots, x_d) = c_\theta(u_1, \dots, u_d) \prod_{i=1}^{d} f_i(x_i)$$

where $u_i = F_{X_i}(x_i)$ and c_θ is the corresponding copula density. The Kullback-Leibler divergence for copulae can be regarded as a distance between two copula densities. It follows from the definition of Kullback-Leibler divergence (for details refer to Spokoiny (2007)):

$$\mathcal{K}(C_{\theta_0}, C_{\theta_1}) = E_{\theta_0}\left[\log\left\{\frac{c_{\theta_0}(U_1, \dots, U_d)}{c_{\theta_1}(U_1, \dots, U_d)}\right\}\right]$$

where $U_i = F_{X_i}(X_i) \sim U[0,1]$ are i.i.d. random variables, $i = 1, \dots, d$. Moreover, for the independence copula $C^{\perp}(u_1, \dots, u_d) = \prod_{i=1}^{d} u_i$ with density $c^{\perp}(u_1, \dots, u_d) = \mathbf{1}_{[0,1]^d}$ it holds:

$$\begin{aligned} \mathcal{K}(C_{\perp}, C_\theta) &= -E_{\perp}[\log c_\theta(U_1, \dots, U_d)] \\ \mathcal{K}(C_\theta, C_{\perp}) &= E_\theta[\log c_\theta(U_1, \dots, U_d)]. \end{aligned}$$

3 Value-at-Risk and Copulae

The *RiskMetrics* VaR procedure assumes that the risk factor X_t have a conditional multivariate normal distribution. For the estimation of Σ_t the covariance matrix of X_t, *RiskMetrics* employs the exponentially weighted moving average model (EWMA). More precisely, the conditional distribution of log-returns is estimated by $N(0, \widehat{\Sigma}_t)$:

$$\widehat{\Sigma}_t = (e^{\lambda} - 1) \sum_{s<t} e^{-\lambda(t-s)} X_s X_s^{\top}.$$

The parameter λ of the model $(0 < \lambda < 1)$ is the so-called *decay factor*, determined by an optimization procedure. The value 0.05, which according to Morgan/Reuters (1996) provides the best backtesting results, is used as the exponential moving average decay factor.

In the copulae based approach one first corrects the contemporaneous volatility in the log-returns process:

$$X_{j,t} = \sigma_{j,t} \varepsilon_{j,t}$$

where $\varepsilon_t = (\varepsilon_{1,t}, \ldots, \varepsilon_{d,t})^{\top}$ are standardised innovations for $j = 1, \ldots, d$ and

$$\sigma_{j,t}^2 = E[X_{j,t}^2 \mid \mathcal{F}_{t-1}]$$

is the conditional variance given $\mathcal{F}_{t-1}$. The innovations $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_d)^{\top}$ have joint distribution F_{ε} and ε_j have continuous marginal distributions F_j, $j = 1, \ldots, d$. The innovations ε have a distribution function described by

$$F_{\varepsilon}(\varepsilon_1, \ldots, \varepsilon_d) = C_{\theta}\{F_1(\varepsilon_1), \ldots, F_d(\varepsilon_d)\}$$

where C_{θ} is a *copula* belonging to a parametric family $\mathcal{C} = \{C_{\theta}, \theta \in \Theta\}$. For details on the above model specification see Chen and Fan (2004), Chen and Fan (2006), Chen et al. (2006). For the Gaussian copula with Gaussian marginals we recover the conditional Gaussian RiskMetrics framework.

To obtain the Value-at-Risk in this set up, the dependence parameter and distribution function from residuals are estimated from a sample of log-returns and used to generate P&L Monte Carlo samples. Their quantiles at different levels are the estimators for the Value-at-Risk, see Embrechts et al. (1999), Bouyé et al. (1996). The whole procedure can be summarized as follows:

For a portfolio $w \in \mathbb{R}^d$ and a sample $\{x_{j,t}\}_{t=1}^T$, $j = 1, \dots, d$ of log-returns, the Value-at-Risk at level α is estimated according to the following steps, see Giacomini and Härdle (2005), Härdle et al. (2002):

1. determination of innovations $\{\hat{\varepsilon}_t\}_{t=1}^T$ by e.g. deGARCHing
2. specification and estimation of marginal distributions $F_j(\hat{\varepsilon}_j)$
3. specification of a parametric copula family $\mathcal{C}$ and estimation of the dependence parameter θ
4. generation of Monte Carlo sample of innovations ε and losses L
5. estimation of $\widehat{VaR}_t(\alpha)$, the empirical α-quantile of F_L.

4 Copula Estimation

Consider a vector of random variables: $X = (X_1, ..., X_d)^\top$ with parametric univariate marginal distributions $F_{X_j}(x_j, \delta_j)$, $j = 1, ..., d$. With (2.3) and $\alpha = (\theta, \delta_1, ..., \delta_d)^\top$ the log-likelihood function is given by:

$$\ell(\alpha; x_1, \dots, x_T) = \sum_{t=1}^{T} \log c\{F_{X_1}(x_{1,t}; \delta_1), \dots, F_{X_d}(x_{d,t}; \delta_d); \theta\} + \sum_{t=1}^{T} \sum_{j=1}^{d} \log f_j(x_{j,t}; \delta_j). \qquad (4.1)$$

The objective is to maximize this log-likelihood. The estimation can be done in three different ways, see Joe (1997), Durrleman et al. (2000). The *full maximum likelihood (FML)* method estimates parameter α in one step through

$$\tilde{\alpha}_{FML} = \arg\max_{\alpha} \ell(\alpha).$$

The drawback of the FML method is that with an increasing scale of the problem the algorithm becomes computationally very burdensome.

In the *inference for margins (IFM)* method for maximizing (4.1) the parameters δ_j are estimated first:

$$\hat{\delta}_j = \arg\max_{\delta} \ell_j(\delta_j)$$

where

$$\ell_j(\delta_j) = \sum_{t=1}^{T} \ln f_j(x_{j,t}; \delta_j)$$

is the log-likelihood function for each of the marginal distributions. The *pseudo log-likelihood* function

$$\ell(\theta, \hat{\delta}_1, \ldots, \hat{\delta}_d) = \sum_{t=1}^{T} \ln c\{F_{X_1}(x_{1,t}; \hat{\delta}_1), \ldots, F_{X_d}(x_{d,t}; \hat{\delta}_d); \theta\}$$

is then maximized over θ to get the dependence parameter estimate $\hat{\theta}$. The IFM is faster and computationally easier to implement.

Canonical Maximum Likelihood (CML) maximizes the *pseudo log-likelihood* function with *empirical* marginal distributions:

$$\ell(\theta) = \sum_{t=1}^{T} \log c\{\widehat{F}_{X_1}(x_{1,t}), \ldots, \widehat{F}_{X_d}(x_{d,t}); \theta\}$$

$$\widehat{\vartheta}_{CML} = \arg\max_{\theta} \ell(\theta)$$

where

$$\widehat{F}_{X_j}(x) = \frac{1}{T+1} \sum_{t=1}^{T} \mathbf{1}\{X_{j,t} \leq x\}.$$

An advantage of the CML over both the other methods is that we do not need to make any assumptions about the parametric form of the marginal distributions. Figure 2 shows that both methods, IFM and CML provide nearly the same estimates for the estimated Clayton copula dependence parameter θ.

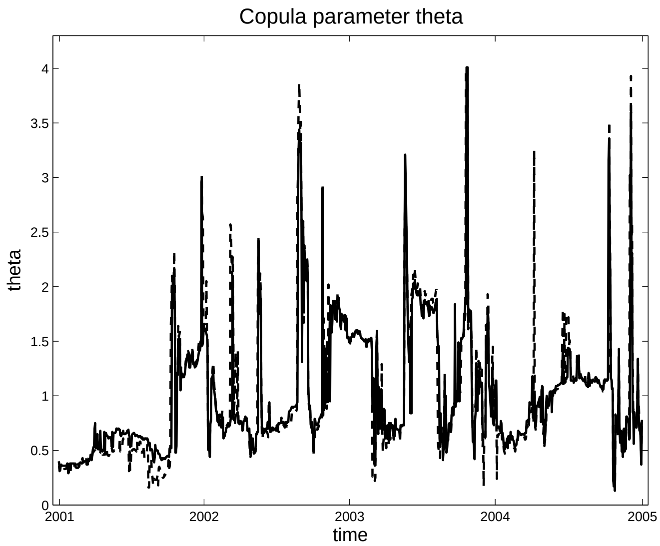


Figure 2: Copula dependence parameter θ estimated using Clayton copula for DaimlerChrysler, Volkswagen, Bayer, BASF, Allianz and Münchener Rückversicherung, 20000101-20041231. Estimated using IFM approach (dashed line) and CML approach (solid line).

5 Inhomogeneous Dependence Modelling with Time Varying Copulae

Very similar to the *Risk Metrics* procedure, one can perform a *moving window* estimation of the copula parameter. This procedure though does not fine tune local changes in dependencies. In fact, the joint distribution F_{t,L_t} from (1.3) is modelled as $F_{t,L_t} = C_{\theta_t}\{F_{t,1}(L_1), \ldots, F_{t,d}(L_d)\}$ with probability measure P_{θ_t}. The moving window of fixed width will estimate a θ_t for each t but will not provide precise estimates close to e.g. a change point in θ_t.

In order to choose an interval of homogeneity we employ a local parametric fitting approach as introduced by Mercurio and Spokoiny (2004) and Härdle et al. (2003). The complete theory is given in Spokoiny (2007). The basic idea is to adaptively estimate an interval of homogeneity in which the hypothesis of a locally constant copula parameter is supported. Using *Local Change*

Point (LCP) detection procedure, see Spokoiny (2007), we sequentially test: θ_t is constant (i.e. $\theta_t = \theta$) within some interval I (local parametric assumption). Thereby we define the "Oracle" choice as the largest interval $I = [t_0 - m_{k^*}, t_0]$, for which the *small modelling bias condition (SMB)*:

$$\Delta_I(\theta) = \sum_{t \in I} \mathcal{K}(P_\theta, P_{\theta_t}) \leq \Delta \tag{5.1}$$

where θ is constant and

$$\mathcal{K}(P_\vartheta, P_{\vartheta'}) = E_\vartheta \log \frac{p(y,\vartheta)}{p(y,\vartheta')}$$

denotes the *Kullback-Leibler divergence*, is fulfilled. The "range point" $t_0 - m_{k^*}$ indicates the largets interval fulfilling (5.1) and θ_{t_0} is ideally estimated from $I = [t_0 - m_{k^*}, t_0]$. The error and risk bounds are calculated in Spokoiny (2007). Other measures of differences between P_θ and P_{θ_t} may be employed. The Kulback-Leibler divergence though is most convenient in our setting since we base our adaptive choice of interval of homogeneity on likelihood ratio theory.

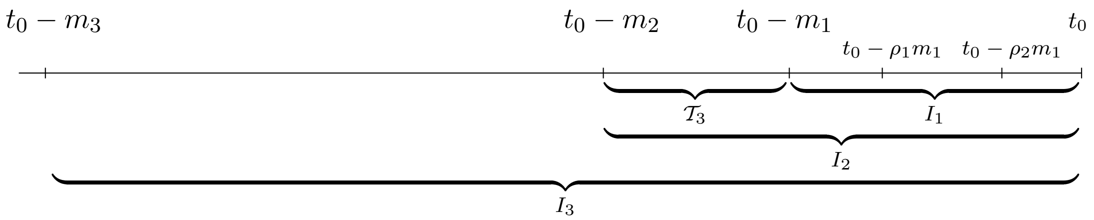
5.1 LCP procedure

The choice of the homogeneity interval is done by the *local change point (LCP)* detection procedure. LCP is based on the adaptive choice of the interval of homogeneity for the endpoint t_0. Defining a family of intervals of the form $\mathcal{I} = \{I_k, k = -1, 0, 1, ...\}$ such that $I_k = [t_0 - m_k, t_0]$ with m_k: $m_{-1} < m_0 < ... \leq t_0$, $m_{-1} = \rho_2 m_1$, $m_0 = \rho_1 m_1$ and $\rho_1 > \rho_2 \in (0,1)$ and defining sets of internal points $\mathcal{T}_k \subset I_k$ of the form $\mathcal{T}_k = [t_0 - m_{k-1}, t_0 - m_{k-2}]$ for $k = 1, 2, \ldots$ we start the procedure with $k = 1$ and

1. test the $H_{0,k}$ hypothesis of homogeneity within I_k on $\mathcal{T}_k$
2. if $H_{0,k}$ is not rejected, take the next larger interval I_{k+1} and repeat the previous step until homogeneity is rejected or the largest possible interval $[0, t_0]$ is reached
3. if $H_{0,k}$ is rejected within I_k, the estimated interval of homogeneity is the last accepted interval

$\widehat{I} = I_{k-2}$

4. if the largest possible interval is reached we take $\widehat{I} = [0, t_0]$.



We estimate the copula dependence parameter θ from observations in $\widehat{I}$, assuming the homogeneous model within $\widehat{I}$, i.e. we define $\widehat{\theta}_{t_0} = \widetilde{\theta}_{\widehat{I}}$. We now describe how to perform the local homogeneity test.

5.1.1 Test of homogeneity against a change point alternative

Let $I = [t_0 - m, t_0]$ be an interval candidate and $\mathcal{T}_I$ be a set of internal points within I. The null hypothesis H_0 means that $\forall \tau \in \mathcal{T}_I$, $\theta_t = \theta$, i.e., the observations in I follow the model with dependence parameter θ. The alternative hypothesis H_1 claims that $\exists \tau \in \mathcal{T}_I$: $\theta_t = \theta_1$ for $t \in J = [\tau, t_0]$ and $\theta_t = \theta_2 \neq \theta_1$ for $t \in J^c = [t_0 - m, \tau[$, i.e. the parameter θ changes spontaneously in some internal point τ of the interval I.

If $\ell_I(\theta)$ and $\ell_J(\theta_1) + \ell_{J^c}(\theta_2)$ are the log-likelihood functions corresponding to H_0 and H_1 respectively, the likelihood ratio test for the single change point with known fixed location τ can be written as:

$$
\begin{aligned}
T_{I,\tau} &= \max_{\theta_1,\theta_2} \{\ell_J(\theta_1) + \ell_{J^c}(\theta_2)\} - \max_{\theta} \ell_I(\theta) \\
&= \ell_J(\hat{\theta}_J) + \ell_{J^c}(\hat{\theta}_{J^c}) - \ell_I(\hat{\theta}_I) \\
&= \hat{\ell}_J + \hat{\ell}_{J^c} - \hat{\ell}_I.
\end{aligned}
$$

The test statistics for unknown change point location is defined as

$$T_I = \max_{\tau \in \mathcal{T}_I} T_{I,\tau}$$

and tests the homogeneity hypothesis in I against the change point alternative with unknown location τ belonging to the set of considered locations $\mathcal{T}_I$. The change point test compares this test statistics with a critical value λ_I which may depend on the interval I and the nominal first kind error probability α. One rejects the hypothesis of homogeneity if $T_I > \lambda_I$. The estimator of the change point is then defined as

$$\widehat{\tau} = \arg\max_{\tau \in \mathcal{T}_{\mathcal{I}}} T_{I,\tau}.$$

5.1.2 Parameters of the LCP procedure

To start the procedure, we have to specify some parameters. This includes: selection of interval candidates $\mathcal{I}$ and internal points $\mathcal{T}_I$ for each of this intervals; choice of the critical values λ_I, which may depend on the interval I and the nominal first kind error probability α. One possible example of an implementation is presented below.

Selection of interval candidates $\mathcal{I}$ and internal points $\mathcal{T}_I$: It is usefull to take the set $\mathcal{I}$ of interval candidates in form of a geometric grid. We fix the length of the interval I_1 to m_1, define

1. $m_0 = \rho_1 m_1$ and $m_{-1} = \rho_2 m_1$ for $\rho_1 > \rho_2 \in (0, 1)$
2. $m_k = [m_1 c^{k-1}]$ for $k = 1, 2, \ldots, K$ and $c > 1$ where $[x]$ means the integer part of x

We set $I_k = [t_0 - m_k, t_0]$ and $\mathcal{T}_k = [t_0 - m_{k-1}, t_0 - m_{k-2}]$ for $k = 1, 2, \ldots, K$

Choice of the critical values λ_I: The event "accept homogeneity in I_{k-1}, reject in I_k" may be represented by the set

$$\mathcal{B}_k = \bigcap_{j=1}^{k-1} \{T_{I_j} \le \lambda_{I_j}\} \cap \{T_{I_k} > \lambda_{I_k}\}$$

and it holds $\mathcal{B}_i \cap \mathcal{B}_j = \emptyset$ for $i \neq j$, $i, j = 1, 2, \ldots$. Thus, defining $\beta_{I_k} = P(\mathcal{B}_k)$ and $\alpha_{I_k} = P\left(\bigcup_{j=1}^{k} \mathcal{B}_j\right)$ we verify

$$\alpha_{I_k} = \sum_{j=1}^{k} \beta_{I_j}$$

The critical values λ_{I_k} are sequentially selected by Monte Carlo simulation to provide, under the homogeneity hypothesis, probability of "false alarm" β_{I_k} for every interval I_k

$$P_{H_0}\left(\bigcap_{j=1}^{k-1} \{T_j \le \lambda_{I_j}\} \cap \{T_{I_k} > \lambda_{I_k}\}\right) = \beta_{I_k}$$

and it follows that α_{I_k} is the probability of at least one false alarm until step k. The standard approach for choosing the critical values is to provide a prescribed first kind error probability $\alpha_K = \alpha$. A reasonable proposal is to set

$$\beta_{I_{K-k+1}} = \alpha m_k^{-1} \left(\sum_{j=1}^{k} m_j^{-1}\right)^{-1}$$

where m_k denotes the number of points in interval I_k.

6 Simulated Examples

6.1 Clayton Copula: sudden jump in dependence

The LCP procedure is applied to different sets of simulations from d-dimensional Clayton copula with parameter given by

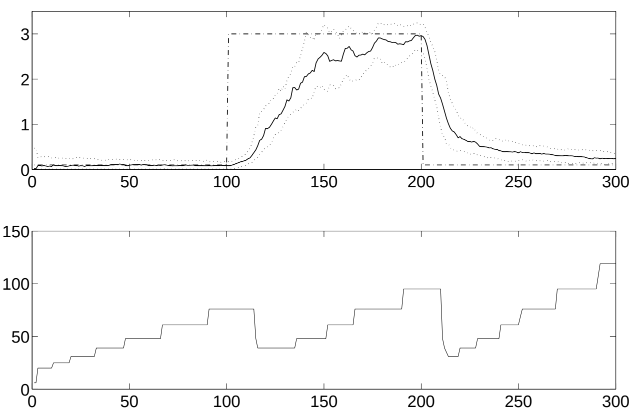


Figure 3: Pointwise median (full), 0.25, 0.75 quantiles (dotted) of estimated parameter $\hat{\theta}_t$, true parameter θ_t (dashed), top. Median of estimated size of homogeneity intervals $|\hat{I}_t|$, bottom. Based on 200 simulations, Clayton copula, $\vartheta = 3$, $d = 2$, $m_1 = 20$ and $c = 1.25$

$$\theta_t = \begin{cases} 0.1 & \text{if} \quad 1 \leq t \leq 100 \\ \vartheta & \text{if} \quad 101 \leq t \leq 200 \\ 0.1 & \text{if} \quad 201 \leq t \leq 300 \end{cases}$$

For each pair of values ϑ and d (for jumps to and from $\vartheta = 1.5, 3$ and 6 and $2-$, $6-$ and $10-$ dimensional copulae), 200 distinct simulations are generated. The dependence parameter and homogeneity intervals are estimated and the detection delay to the jumps computed for each of the sets. Figures 3, 4 and 6 show the pointwise median and quantiles of the estimated parameter $\hat{\theta}_t$ and pointwise median of the size of estimated homogeneity intervals $|\hat{I}_t|$.

The *detection delay* δ at rule $r \in [0, 1]$ to jump of size $\Delta = \theta_t - \theta_{t-1}$ and $t \in \{101, 201\}$ is expressed by

$$\delta(t, \Delta, r) = \delta^* \mathbf{1}_{\{\delta^* < 100\}} + (100)\mathbf{1}_{\{\delta^* \geq 100\}}$$

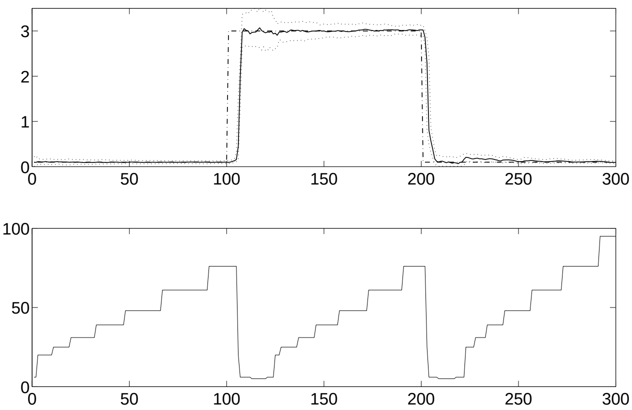


Figure 4: Pointwise median (full), 0.25, 0.75 quantiles (dotted) of estimated parameter $\hat{\theta}_t$, true parameter θ_t (dashed), top. Median of estimated size of homogeneity intervals $|\hat{I}_t|$, bottom. Based on 200 simulations, Clayton copula, $\vartheta = 3$, $d = 6$, $m_1 = 20$ and $c = 1.25$

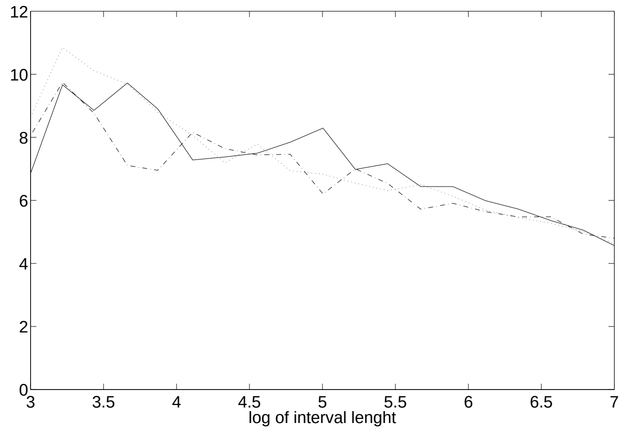


Figure 5: Critical values λ_{I_k} for $\alpha = 0.05$, $m_1 = 20$, $c = 1.25$, $d = 2$ (dotted), 6 (dashed) and 10 (full)

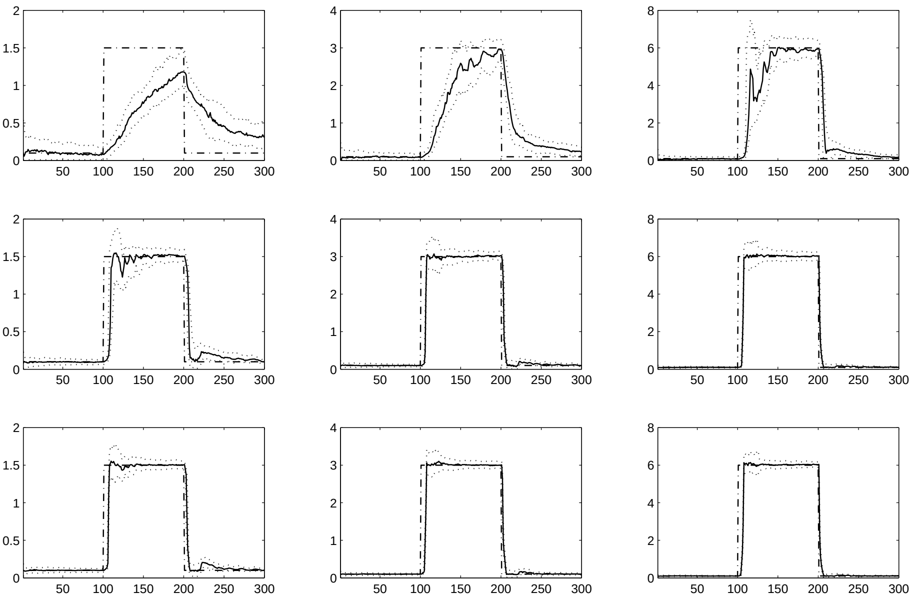


Figure 6: Pointwise median (full), 0.25, 0.75 quantiles (dotted) from estimated parameter $\hat{\theta}_t$ and true parameter θ_t (dashed), from left to right $\vartheta = 1.5, 3, 6$, from top to bottom $d = 2, 6, 10$. Based on 200 simulations from Clayton copula, $m_1 = 20$ and $c = 1.25$

where

$$\delta^* = \min\{k \geq t : \hat{\theta}_k = \theta_{t-1} + r\Delta\} - t$$

and $\hat{\theta}_t$ is the estimated parameter at t. It represents the number of steps necessary for the estimated parameter to reach the r-fraction of a jump in the real parameter (if the fraction is not reached in 100 steps, the delay is set to 100).

Detection delays are proportional to probability of error of type II, i.e., probability of accepting homogeneity in case of jump. Thus, tests with higher power correspond to lower detection delays. The Kullback-Leibler divergences for upward ($\mathcal{K}_d(0.1, \vartheta)$) and downward ($\mathcal{K}_d(\vartheta, 0.1)$) jumps for d-dimensional Clayton copulae are proportional to the power of the respective homogeneity tests and are displayed in table 1. We verify that for Clayton copulae the divergence is increasing in the size of jump and in dimension and is also higher for upward than for downward jumps (fig. 8)

The descriptive statistics for detection delays to jumps at $t = 101$ and 102 are in table 1. The mean detection delay decreases with ϑ and dimension d. Moreover they are higher for downward jumps (at $t = 101$) than for upward (at $t = 102$). Figure 7 displays the logarithm of mean detection delay against jump size for $r = 0.6$ for upward and downward jumps and respective dimensions.

6.2 Clayton Copula: linear change in dependence

The procedure is applied on simulated data with linear increase and decrease in dependence. Similarly to the last section, different sets of simulations from d-dimensional Clayton copula with parameter given by

			$\vartheta = 1.5$				$\vartheta = 3$				$\vartheta = 6$			
d	t	r	mean	std dev.	max	min	mean	std dev.	max	min	mean	std dev.	max	min
2	101	40%	35.64	16.49	93	1	23.05	9.41	62	6	14.31	6.39	38	4
		50%	41.70	19.23	100	2	26.34	11.79	67	6	15.24	7.60	41	4
		60%	50.04	21.93	100	7	29.31	13.82	84	6	16.00	8.62	41	4
	201	40%	9.27	10.39	62	1	8.27	5.79	30	1	5.55	2.43	16	1
		50%	14.70	13.38	74	1	10.62	6.20	32	1	6.07	2.73	17	1
		60%	25.78	21.04	100	1	12.87	7.30	45	1	6.66	3.21	22	1
6	101	40%	8.84	3.53	31	2	5.82	1.07	9	2	6.22	0.80	7	4
		50%	9.35	4.39	34	2	6.07	0.97	9	3	6.43	0.68	7	4
		60%	10.00	5.57	34	3	6.32	0.92	9	4	6.62	0.61	9	5
	201	40%	5.33	2.69	14	1	2.89	1.39	7	1	1.61	0.74	4	1
		50%	5.87	3.13	15	1	3.01	1.45	7	1	1.62	0.75	4	1
		60%	6.31	3.57	20	1	3.07	1.46	7	1	1.74	0.81	4	1
10	101	40%	5.84	1.57	13	2	5.67	1.00	7	2	6.34	0.68	7	4
		50%	6.04	1.51	13	2	6.04	0.91	7	3	6.60	0.55	7	5
		60%	6.26	1.40	13	3	6.37	0.75	7	4	6.68	0.52	7	5
	201	40%	3.61	1.68	10	1	2.01	0.90	4	1	1.24	0.46	3	1
		50%	3.69	1.72	10	1	2.07	0.95	4	1	1.26	0.49	3	1
		60%	3.79	1.71	10	1	2.31	1.06	5	1	1.51	0.66	3	1

Table 1: Statistics for detection delay δ to downward ($t = 101$) and upward ($t = 201$) jump of size $\vartheta - 0.1$ at rule r, based on 200 simulations from d-dimensional Clayton copula, $m_1 = 20$, $c = 1.25$

ϑ	$\mathcal{K}_2(0.1, \vartheta)$	$\mathcal{K}_2(\vartheta, 0.1)$	$\mathcal{K}_6(0.1, \vartheta)$	$\mathcal{K}_6(\vartheta, 0.1)$	$\mathcal{K}_{10}(0.1, \vartheta)$	$\mathcal{K}_{10}(\vartheta, 0.1)$
1.5	0.41	0.26	3.52	1.57	7.30	2.89
3.0	1.28	0.56	11.49	3.25	24.69	5.89
6.0	3.51	1.01	31.52	5.56	68.35	10.00

Table 2: Kullback-Leibler divergence between d-dimensional Clayton copulae with parameters 0.1 and ϑ

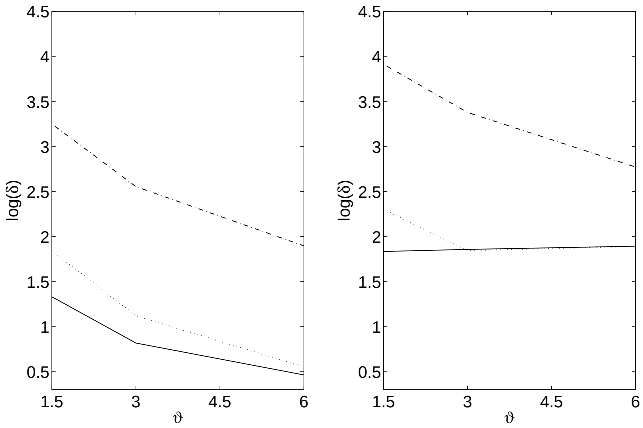


Figure 7: Logarithm of mean detection delays at $r = 0.6$ for different upward (left) and downward (right) jump sizes, d-dimensional Clayton Copula, $d = 2$ (dashed), 6 (dotted) and 10 (full)

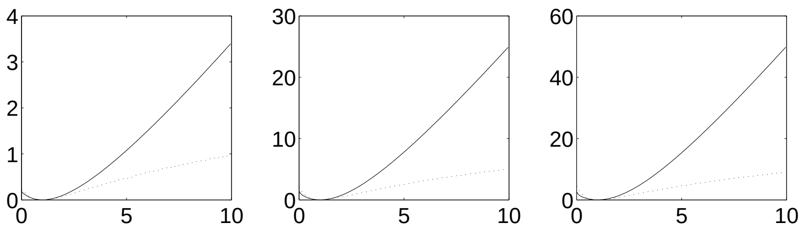


Figure 8: $\mathcal{K}_d(0.1, \vartheta)$ (dashed), $\mathcal{K}_d(\vartheta, 0.1)$ (full), corresponding to upward and downward jumps, d-dimensional Clayton copula, $d = 2$ (left), 6 (middle) and 10 (right)

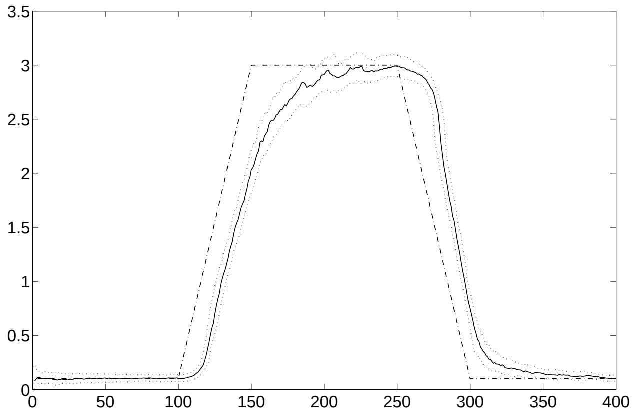


Figure 9: Pointwise median (full), 0.25, 0.75 quantiles (dotted) from estimated parameter $\hat{\theta}_t$ and true parameter θ_t (dashed), $\vartheta = 3$, $d = 6$. Based on 200 simulations from Clayton copula, $m_1 = 20$ and $c = 1.25$

$$\theta_t = \begin{cases} 0.1 & \text{if} \quad 1 \leq t \leq 100 \\ 0.1 + \frac{1}{50}\Delta(t - 100) & \text{if} \quad 101 \leq t \leq 150 \\ \vartheta & \text{if} \quad 151 \leq t \leq 250 \\ \vartheta - \frac{1}{50}\Delta(t - 250) & \text{if} \quad 251 \leq t \leq 300 \\ 0.1 & \text{if} \quad 301 \leq t \leq 400 \end{cases}$$

and $\Delta = \vartheta - 0.1$ are generated. Figures 9 and 10 depict the pointwise median and quantiles of the estimated parameter $\hat{\theta}_t$ and the true parameter θ_t for $\vartheta = 1.5, 3$ and 6 and $2-$, $6-$ and $10-$dimensional copulae.

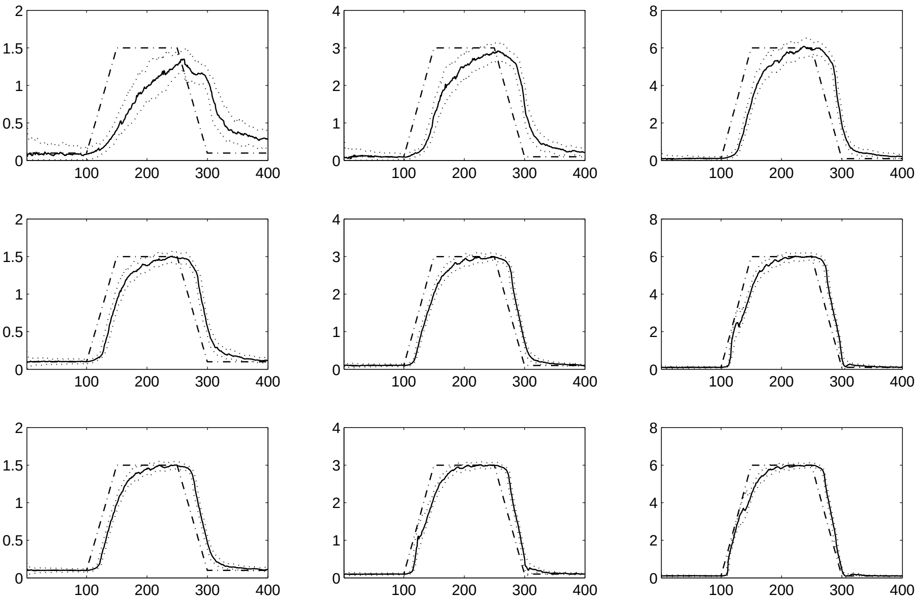


Figure 10: Pointwise median (full), 0.25, 0.75 quantiles (dotted) of estimated parameter $\hat{\theta}_t$, true parameter θ_t (dashed), from left to right $\vartheta = 1.5, 3, 6$, from top to bottom $d = 2, 6, 10$. Based on 200 simulations from Clayton copula, $m_1 = 20$ and $c = 1.25$

t	r	mean	std dev.	max	min
100	40%	18.11	7.15	43	6
	50%	19.69	7.74	43	6
	60%	22.24	9.42	46	8
200	40%	16.02	9.08	45	2
	50%	20.42	13.19	63	2
	60%	25.21	18.16	100	2

Table 3: Statistics for detection delay δ to downward ($t = 101$) and upward ($t = 201$) jump of size 0.8 at rule r, based on 100 simulations from Gaussian copula, $m_1 = 20$, $c = 1.25$

6.3 Gaussian Copula: sudden jump in correlation

The Gaussian copula is parametrized by its correlation matrix (2.4). In the 2 dimensional Gaussian copula the parameter is the correlation coefficient ρ. As in previous sections, the LCP procedure is applied to sets of simulations from 2-dimensional Gaussian copula with parameter given by

$$\rho_t = \begin{cases} 0 & \text{if} \quad 1 \le t \le 100 \\ \varrho & \text{if} \quad 101 \le t \le 200 \\ 0 & \text{if} \quad 201 \le t \le 300 \end{cases}$$

Figure 11 shows the pointwise median and quantiles of the estimated parameter $\hat{\rho}_t$ and the true parameter ρ_t for $\varrho = 0.8$. The Kullback-Leibler divergences corresponding to up and downward jumps in the 2-dimensional Gaussian copula are displayed in fig. 12 as a function of ϱ. For $\varrho = 0.8$ the divergences are $\mathcal{K}_2(0, 0.8) = 1.78$ and $\mathcal{K}_2(0.8, 0) = 0.51$. The detection delay statistics for sudden jump in correlation for Gaussian copula at rule 60% are depicted in table 3.

In the 3-dimensional case the parameter is the correlation matrix Ψ. The LCP procedure is applied to sets of simulations from a 3-dimensional Gaussian copula with correlation given by

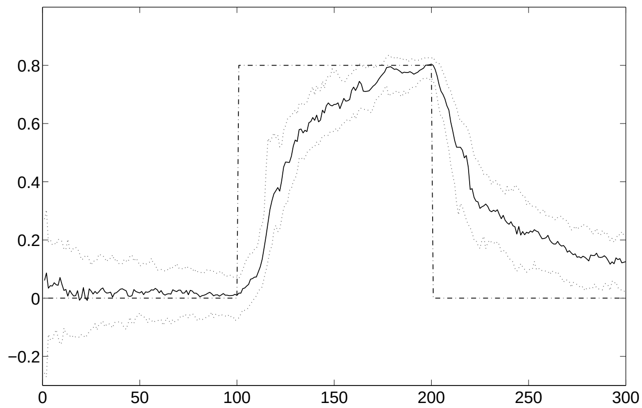


Figure 11: Pointwise median (full), 0.25, 0.75 quantiles (dotted) from estimated parameter $\hat{\rho}_t$ and true parameter ρ_t (dashed), $\varrho = 0.8$. Based on 100 simulations from Gaussian copula, $m_1 = 20$ and $c = 1.25$

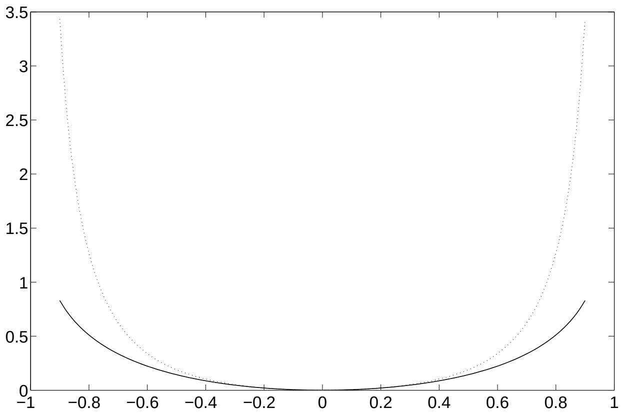


Figure 12: $\mathcal{K}_2(0, \varrho)$ (dashed), $\mathcal{K}_2(\varrho, 0)$ (full), corresponding to upward and downward jumps, 2-dimensional Gaussian copula

$$\Psi_t = \begin{cases} \mathcal{I}_3 & \text{if} \quad 1 \leq t \leq 100 \\ \mathcal{R} & \text{if} \quad 101 \leq t \leq 200 \\ \mathcal{I}_3 & \text{if} \quad 201 \leq t \leq 300 \end{cases}$$

where $\mathcal{I}_3$ is the identity matrix of size 3 and

$$\mathcal{R} = \begin{pmatrix} 1 & 0.8 & 0 \\ 0.8 & 1 & -0.5 \\ 0 & -0.5 & 1 \end{pmatrix}$$

The distance between the estimated $\hat{\Psi}$ and the true correlation matrix Ψ, $d(\hat{\Psi}, \Psi)$ is given by

$$d(\hat{\Psi}, \Psi) = ||\hat{\psi} - \psi||_2 \tag{6.1}$$

where $\psi = (\psi_{12}, \psi_{13}, \psi_{23})^\top$ and ψ_{ij} is the (i, j) element of matrix Ψ. This distance is motivated by the *Frobenius* norm for a matrix $\mathcal{A}$, $||\mathcal{A}||_F = \left(\sum_{i,j} |a_{ij}|^2\right)^{\frac{1}{2}}$ and we have $d(\mathcal{R}, \mathcal{I}_3) = 0.9434$. Figure 13 depicts the pointwise median and quantiles of distance $d(\hat{\Psi}_t, \Psi_t)$ between estimated and true correlation matrices.

7 Empirical Results

The estimation methods described in the preceeding section (*RiskMetrics*, moving window and adaptive estimation procedure) are applied to a portfolio composed of two different sets of DAX stocks. At first we apply the procedure to DaimlerChrysler (DCX), Volkswagen (VW), Allianz (ALV), Münchener Rückversicherung (MUV2), Bayer (BAY) and BASF (BAS) and afterwards to Siemens (SIE), ThyssenKrupp (THY), Schering (SCH), E.ON AG (EOA), Henkel (HEN) and Lufthansa (LHA). The observation period for both data sets covers January 1st to December 31st, 2004 (data available in `http://sfb649.wiwi.hu-berlin.de/fedc`). For the log-returns $\{X_{j,t}\}$

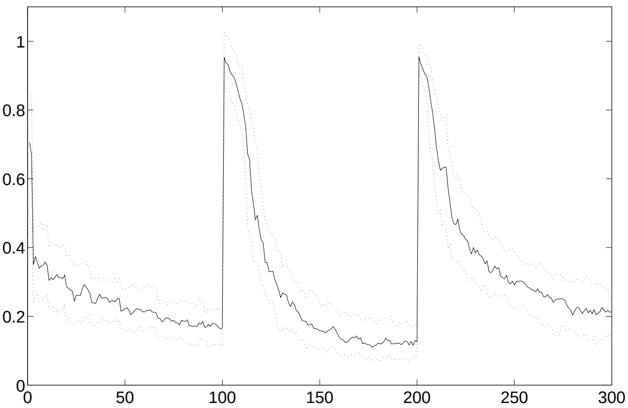


Figure 13: Pointwise median (full), 0.25, 0.75 quantiles (dotted) of distance $d(\hat{\Psi}_t, \Psi_t)$ between estimated and true correlation matrices. Based on 200 simulations from Gaussian copula, $d = 3$, $m_1 = 20$ and $c = 1.25$

modelled as

$$X_{j,t} = \sigma_{j,t}\varepsilon_{j,t}$$

we estimate the parameters $\sigma^2_{j,t}$ using exponential smoothing techniques for every time point t:

$$\hat{\sigma}^2_{j,t} = (e^{\lambda} - 1)\sum_{s<t} e^{-\lambda(t-s)} X^2_{s,j}$$

where $X_{j,s}$, $j = 1, ..., 6$ denotes log returns of DCX, VW, ALV, MUV, BAY and BAS (SIE, THY, SCH, EOA, HEN and LHA) at time point s (we set $\lambda = 1/20$).

The chosen copula belongs to the Clayton family since it allows to capture the dependence in the lower tail which is essential for VaR calculation. For the moving window approach we fix $w = 250$; for the LCP procedure we set $\alpha = 0.05$, $c = 1.25$ and $m_1 = 20$. We have chosen these parameters from our experience in simulations. For details on robustness of the reported results with respect to the choice of the parameters c, ρ_1, ρ_2 and m_1 refer to Spokoiny (2007).

The performance of VaR estimation is evaluated based on backtesting. The estimated values for the VaR are compared with the true realizations $\{l_t\}$ of the P&L function, an *exceedance* occuring for each l_t smaller than $\widehat{VaR}_t(\alpha)$. The ratio of the number of exceedances to the number of observations gives the *exceedances ratio* $\hat{\alpha}$:

$$\hat{\alpha} = \frac{1}{T - w} \sum_{t=w}^{T} \mathbf{1}_{\{l_t < \widehat{VaR}_t(\alpha)\}}$$

7.1 DCX, VW, ALV, MUV, BAY and BAS

At first, we analyze the performance of the procedure by applying it to the first portfolio of the DAX stocks: DCX, VW, ALV, MUV, BAY and BAS. Figures 14 and 15 represent the copula dependence parameter and the intervals of homogeneity estimated with the parameters $m_1 = 20$ and $m_1 = 50$, respectively. From the aforementioned Figures in combination with Figure 16 we can observe that with increasing m_1 the estimated copula parameter θ takes on smaller values and its peaks diminish. Accordingly, the intervals of homogeneity become smoother. Further, the analysis shows a September 11 effect: before the terror attack the copula parameter experienced small fluctuations below the value of the global parameter. At the same time, the lengths of the intervals of homogeneity reached high levels. After the attack, the dependence among the stocks becomes larger and the lengths of intervals of homogeneity increase.

The results of the VaR estimation are summarized in Table 4 for Riskmetrics, in Table 5 for the moving window and in Table 6 for the adaptive estimation procedure. They represent exceedance ratios at different levels $\alpha = 1\%$ to $\alpha = 5\%$, at which the VaR has been calculated. Further, the absolute and the relative sum of squared deviations of the exceedance $\hat{\alpha}$ from the actual level α are calculated. We can observe that Riskmetrics outperforms the moving window and the adaptive estimation procedures for higher quantiles: relative squared deviation $\sum_{w \in W} (\hat{\alpha} - \alpha)^2 / \alpha$ for Riskmetrics accounts to 13.34, 17.01 and 26.55 at 5%, 4% and 3% levels respectively, whereas for the moving window and the LCP approach we observe values between 19.67 at 5% level and 30.5 at 3% level (see Table 7). However, Riskmetrics fails to capture the lower tail dependence while

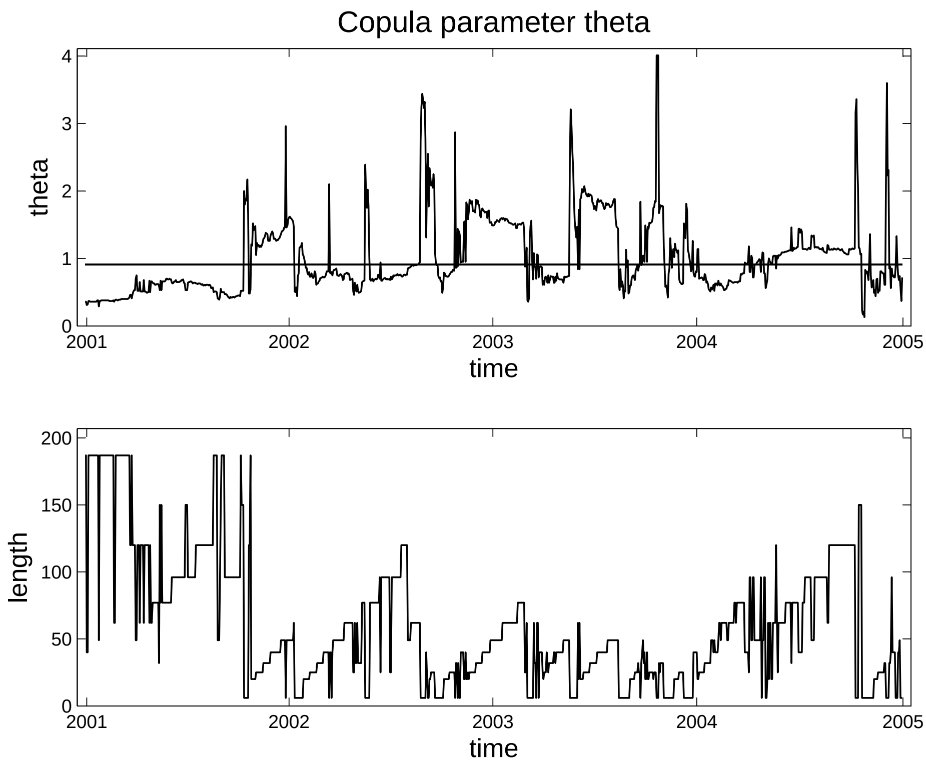


Figure 14: Upper panel: estimated copula dependence parameter θ for 6-dim data: Daimler-Chrysler, Volkswagen, Bayer, BASF, Allianz and Münchener Rückversicherung and the global parameter. Lower panel: estimated intervals of time homogeneity; with parameters $m_1 = 20$, $c = 1.25$ and $\alpha = 0.05$.

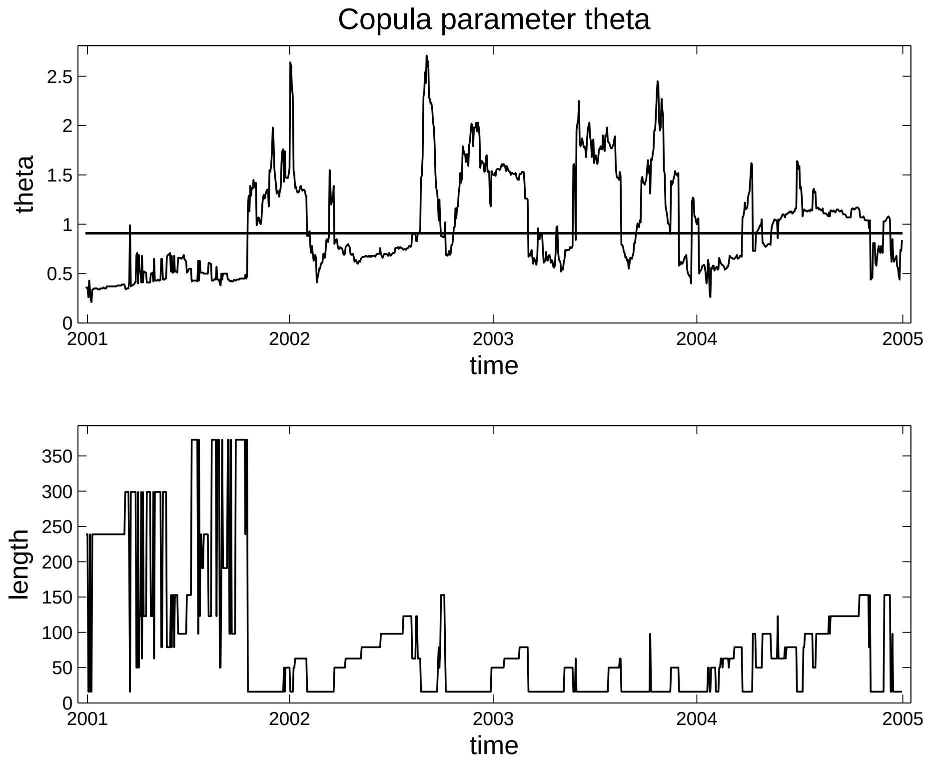


Figure 15: Upper panel: estimated copula dependence parameter θ for 6-dim data: Daimler-Chrysler, Volkswagen, Bayer, BASF, Allianz and Münchener Rückversicherung and the global parameter. Lower panel: estimated intervals of time homogeneity; with parameters $m_1 = 50$, $c = 1.25$ and $\alpha = 0.05$.

	Exceedances ratio $\alpha(\times 10^2)$				
Portfolio	5.00	4.00	3.00	2.00	1.00
$(1,1,1,1,1,1)$	6.77	5.88	4.90	4.02	3.43
$(1,2,3,2,1,3)$	6.86	5.69	4.61	4.12	3.33
$(2,1,2,3,1,3)$	5.59	5.00	4.51	3.63	2.84
$(3,2,3,2,3,1)$	7.16	5.98	5.10	4.31	3.43
$(3,1,2,1,3,2)$	7.94	7.16	5.79	5.00	3.63
$(1,3,1,2,3,1)$	6.47	5.59	4.61	4.21	2.94
$(2,1,3,2,1,3)$	6.67	5.49	4.61	4.12	3.43
$(2,3,3,2,1,1)$	6.96	5.79	4.90	4.12	3.53
$(3,1,2,2,2,3)$	6.77	5.88	4.90	3.92	3.43
$(2,3,1,1,2,3)$	8.24	7.16	5.79	4.51	3.63
$(2,3,2,3,2,3)$	6.18	5.59	4.61	3.92	2.84
$(3,2,3,2,3,3)$	7.26	6.47	5.39	4.31	3.53
$(1,1,1,1,1,-1)$	5.39	4.80	4.41	3.82	3.04
$(1,2,3,2,1,-3)$	5.00	4.41	4.31	3.53	2.64
$(2,1,2,3,1,-3)$	4.41	4.21	3.63	3.14	2.15
$(3,2,3,2,3,-1)$	6.86	5.79	4.90	4.12	3.53
$(3,1,2,1,3,-2)$	7.55	6.28	5.10	4.41	3.72
$(1,3,1,2,3,-1)$	5.69	4.80	4.51	3.92	2.74
$(2,1,3,2,1,-3)$	5.10	4.51	4.31	3.43	2.64
$(2,3,3,2,1,-1)$	6.47	5.29	4.71	4.02	3.43
$(3,1,2,2,2,-3)$	5.00	4.51	4.41	3.63	2.84
$(2,3,1,1,2,-3)$	6.47	5.69	5.29	4.21	3.04
$(2,3,2,3,2,-3)$	4.80	4.41	4.12	3.43	2.74
$(3,2,3,2,3,-3)$	6.37	5.10	4.61	3.92	3.23
avg.	6.33	5.48	4.75	3.99	3.16
std.dev.	1.01	0.81	0.49	0.40	0.40
$\sum_{w\in W}(\hat{\alpha}-\alpha)^2$	0.66	0.68	0.79	0.99	1.15
$\sum_{w\in W}(\hat{\alpha}-\alpha)^2/\alpha$	13.34	17.00	26.54	49.62	115.89

Table 4: Exceedances ratio $\hat{\alpha}$ for different portfolios, estimated using *RiskMetrics* approach for 6-dim data: DCX, VW, ALV, MUV, BAY and BAS.

Portfolio	Exceedances ratio $\alpha(\times 10^2)$				
	5.00	4.00	3.00	2.00	1.00
$(1, 1, 1, 1, 1, 1)$	7.06	6.08	4.80	3.43	1.76
$(1, 2, 3, 2, 1, 3)$	7.36	6.28	4.80	3.72	1.76
$(2, 1, 2, 3, 1, 3)$	7.45	6.37	4.80	3.63	1.37
$(3, 2, 3, 2, 3, 1)$	7.45	6.28	4.80	3.53	1.96
$(3, 1, 2, 1, 3, 2)$	6.67	5.69	4.71	3.33	1.86
$(1, 3, 1, 2, 3, 1)$	6.47	5.59	4.12	3.04	1.66
$(2, 1, 3, 2, 1, 3)$	7.36	6.28	4.80	3.82	1.76
$(2, 3, 3, 2, 1, 1)$	7.55	6.28	5.00	3.63	1.96
$(3, 1, 2, 2, 2, 3)$	6.96	6.08	4.90	3.82	1.86
$(2, 3, 1, 1, 2, 3)$	6.47	5.39	4.31	3.04	1.76
$(2, 3, 2, 3, 2, 3)$	7.06	6.08	4.61	3.43	1.57
$(3, 2, 3, 2, 3, 3)$	7.06	6.08	4.80	3.33	1.86
$(1, 1, 1, 1, 1, -1)$	7.06	6.18	5.59	3.72	1.66
$(1, 2, 3, 2, 1, -3)$	7.75	6.67	5.29	4.21	1.86
$(2, 1, 2, 3, 1, -3)$	7.65	6.67	5.49	4.41	1.57
$(3, 2, 3, 2, 3, -1)$	7.16	6.37	4.80	3.82	1.66
$(3, 1, 2, 1, 3, -2)$	7.36	6.08	4.90	4.02	1.96
$(1, 3, 1, 2, 3, -1)$	6.86	5.98	4.31	3.14	1.37
$(2, 1, 3, 2, 1, -3)$	7.85	6.67	5.39	4.02	1.86
$(2, 3, 3, 2, 1, -1)$	7.65	6.08	5.10	4.02	1.76
$(3, 1, 2, 2, 2, -3)$	7.65	6.37	5.39	3.92	1.57
$(2, 3, 1, 1, 2, -3)$	6.86	5.88	4.90	3.04	1.37
$(2, 3, 2, 3, 2, -3)$	7.06	6.28	5.10	3.92	1.66
$(3, 2, 3, 2, 3, -3)$	7.55	6.28	5.20	4.02	1.57
avg.	7.22	6.17	4.91	3.67	1.71
std.dev.	0.38	0.31	0.36	0.38	0.18
$\sum_{w\in W}(\hat{\alpha}-\alpha)^2$	1.22	1.15	0.91	0.70	0.12
$\sum_{w\in W}(\hat{\alpha}-\alpha)^2/\alpha$	24.55	28.83	30.50	35.23	12.96

Table 5: Exceedances ratio $\hat{\alpha}$ for different portfolios, estimated with Clayton copula using moving window approach for 6-dim data: DCX, VW, ALV, MUV, BAY and BAS.

Portfolio	Exceedances ratio $\alpha(\times 10^2)$				
	5.00	4.00	3.00	2.00	1.00
$(1,1,1,1,1,1)$	7.06	6.08	4.80	2.84	1.57
$(1,2,3,2,1,3)$	7.06	6.37	4.71	3.14	1.76
$(2,1,2,3,1,3)$	7.06	6.28	4.90	3.04	1.37
$(3,2,3,2,3,1)$	7.16	6.08	5.00	3.33	1.76
$(3,1,2,1,3,2)$	6.57	5.79	4.41	3.33	1.76
$(1,3,1,2,3,1)$	6.18	5.59	4.41	2.94	1.57
$(2,1,3,2,1,3)$	7.16	6.28	4.61	3.33	1.76
$(2,3,3,2,1,1)$	7.06	6.47	5.10	3.43	1.66
$(3,1,2,2,2,3)$	6.96	5.88	4.71	3.04	1.66
$(2,3,1,1,2,3)$	6.28	5.20	4.61	2.84	1.86
$(2,3,2,3,2,3)$	6.77	5.88	4.80	2.94	1.37
$(3,2,3,2,3,3)$	6.96	6.08	4.90	3.14	1.86
$(1,1,1,1,1,-1)$	7.26	6.18	5.49	3.33	1.37
$(1,2,3,2,1,-3)$	7.26	6.37	5.20	3.63	1.66
$(2,1,2,3,1,-3)$	7.75	6.37	4.90	4.02	1.66
$(3,2,3,2,3,-1)$	7.26	6.08	5.10	3.72	1.57
$(3,1,2,1,3,-2)$	6.77	5.69	4.90	3.53	1.57
$(1,3,1,2,3,-1)$	6.47	5.49	4.31	3.04	1.37
$(2,1,3,2,1,-3)$	7.65	6.18	5.39	3.82	1.57
$(2,3,3,2,1,-1)$	7.36	6.28	5.00	3.82	1.47
$(3,1,2,2,2,-3)$	7.26	6.08	5.29	3.53	1.47
$(2,3,1,1,2,-3)$	5.98	5.69	4.90	3.14	1.17
$(2,3,2,3,2,-3)$	6.96	6.18	4.90	3.33	1.37
$(3,2,3,2,3,-3)$	7.16	6.08	5.00	3.63	1.37
avg.	6.97	6.03	4.89	3.33	1.57
std.dev	0.43	0.31	0.29	0.33	0.18
$\sum_{w\in W}(\hat{\alpha}-\alpha)^2$	0.98	1.01	0.88	0.45	0.08
$\sum_{w\in W}(\hat{\alpha}-\alpha)^2/\alpha$	19.6	25.33	29.38	22.57	8.57

Table 6: Exceedances ratio $\hat{\alpha}$ for different portfolios, estimated with Clayton copula using adaptive estimation procedure for 6-dim data: DCX, VW, ALV, MUV, BAY and BAS.

Method	Exceedances ratio $\alpha(\times 10^2)$				
	5	4	3	2	1
Riskmetrics	13.34	17.00	26.54	49.62	115.89
Moving Window	24.55	28.83	30.50	35.23	12.96
LCP	19.66	25.33	29.38	22.57	8.57

Table 7: Relative squared deviation $\sum_{w\in W}(\hat{\alpha}-\alpha)^2/\alpha$ for Riskmetrics, Moving Window and LCP approach (DCX, VW, ALV, MUV, BAY and BAS).

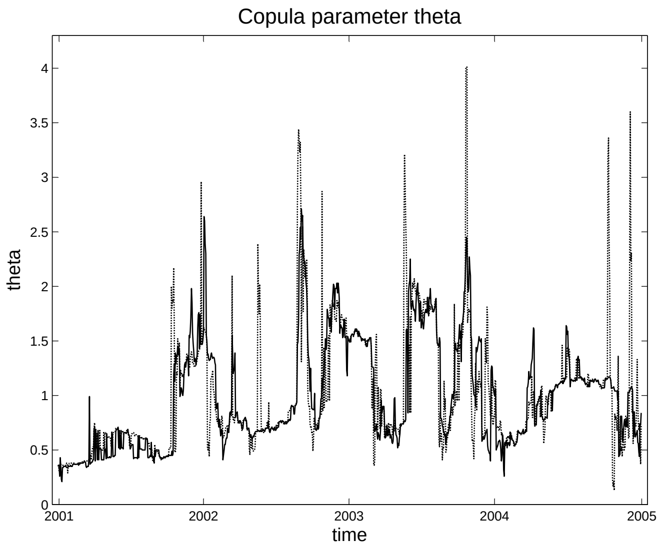


Figure 16: Estimated copula dependence parameter θ for 6-dim data: DaimlerChrysler, Volkswagen, Bayer, BASF, Allianz and Münchener Rückversicherung estimated with parameter $m_1 = 20$ (dashed line) and $m_1 = 50$ (solid line).

copula-based approaches provide better results: for example, the relative squared deviation at the 1% level is in case of Riskmetrics is at least 10 times as high as for the adaptive procedure. Further, the exceedances ratios in Riskmetrics case are more volatile: the standard deviations account to 1.02 to 4.03 for Riskmetrics, whereas with a copula-based approch we obtain values between 0.18 and 0.43 .

7.2 SIE, THY, SCH, EOA, HEN and LHA

We consider now a portfolio consisting of the DAX stocks SIE, THY, SCH, EOA, HEN and LHA. The copula dependence parameter and the intervals of homogeneity estimated with parameters $m_1 = 20$ and $m_1 = 50$ are plotted in Figures 18 and 19 respectively. As in the case of DCX, VW, ALV, MUV, BAY and BAS, with increasing m_1 we observe diminishing of peaks in the estimated values of copula dependence parameter (Figure 20) and smoother pattern for the length of the

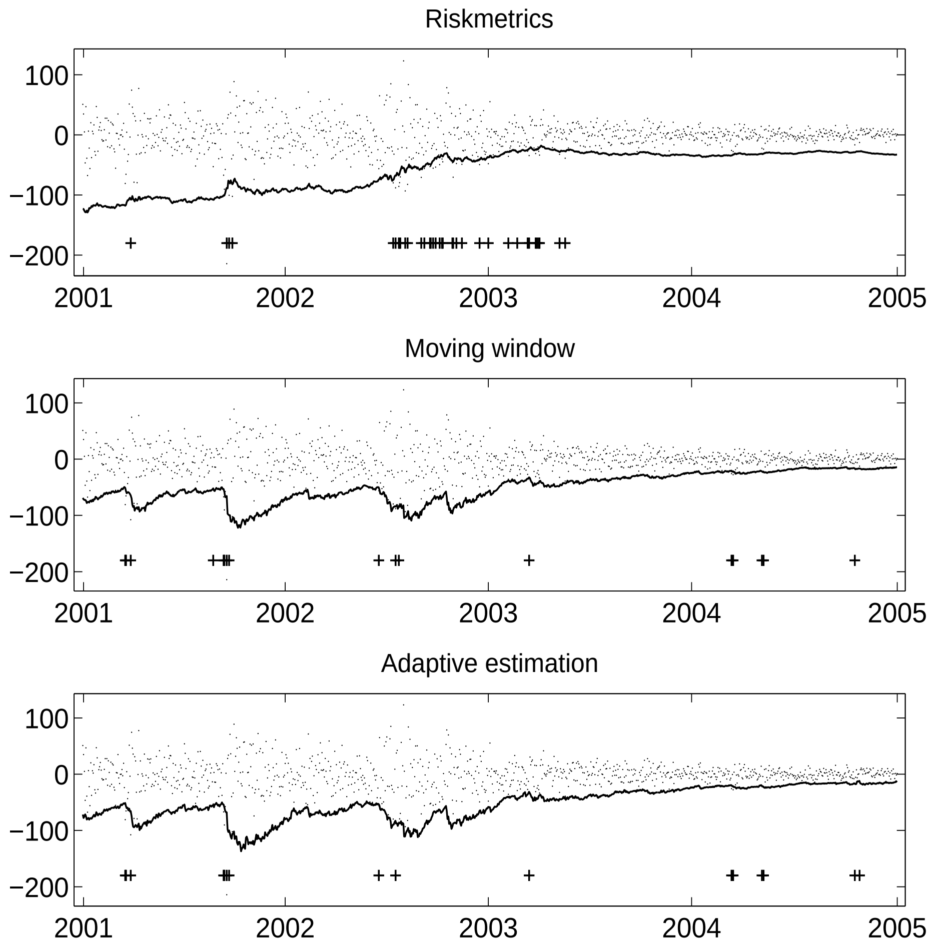


Figure 17: P&L (dots) and $\widehat{VaR}(\alpha)$ at level $\alpha_1 = 0.01$; $w = (3, 2, 3, 2, 3, -1)^\top$, estimated using *RiskMetrics* approach (upper panel), moving window approach (middle panel) and adaptive estimation procedure (lower panel) for 6-dim data: DaimlerChrysler, Volkswagen, Bayer, BASF, Allianz and Münchener Rückversicherung.

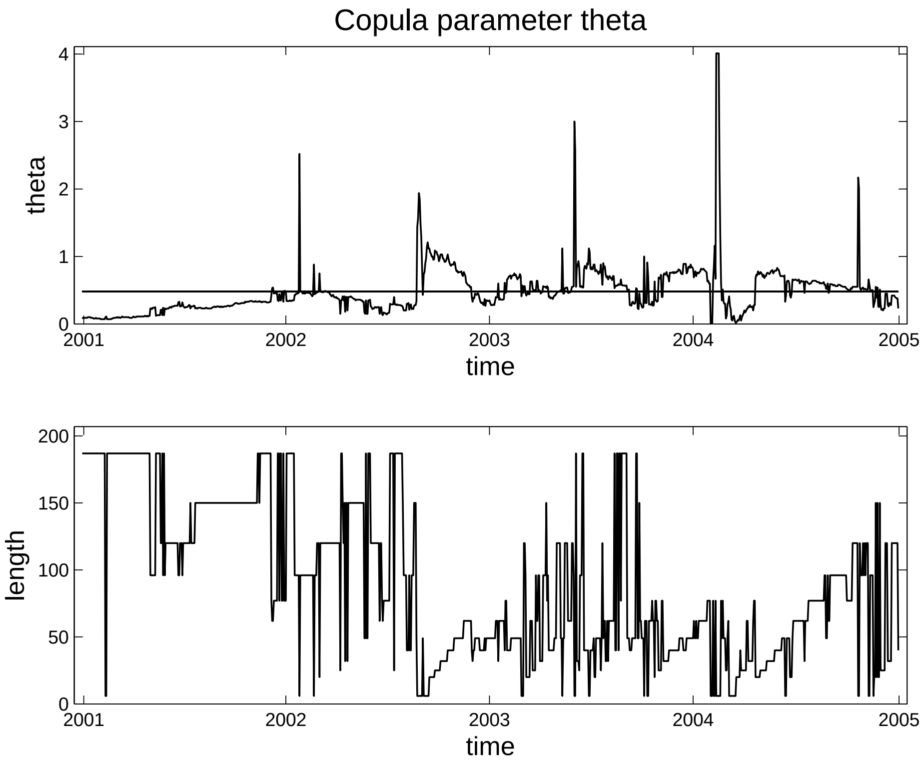


Figure 18: Upper panel: estimated copula dependence parameter θ for 6-dim data: Siemens, ThyssenKrupp, Schering, E.ON AG, Henkel, Lufthansa and the global parameter. Lower panel: estimated intervals of time homogeneity; with parameters $m_1 = 20$, $c = 1.25$ and $\alpha = 0.05$.

intervals of homogeneity.

The results of the VaR estimation are summarized in Table 8 for Riskmetrics, in Table 9 for the moving window and in Table 10 for the adaptive estimation procedure. We can observe that at the 5% level Riskmetrics performs better than moving window. However, the adaptive procedure produces even better results: the relative squared deviations acount to 6.96, 7.38 and 3.59 for Riskmetrics, moving window and LCP procedure, respectively (Table 11). For the quantiles at levels $\alpha = 4\%$ to $\alpha = 1\%$ copula-based approaches outperform Riskmetrics, whereas the adaptive procedure leads to the smallest values of the relative squared deviations: taking on values between 3.53 and 7.42, it produces results twice as good as moving window and Riskmetrics approaches.

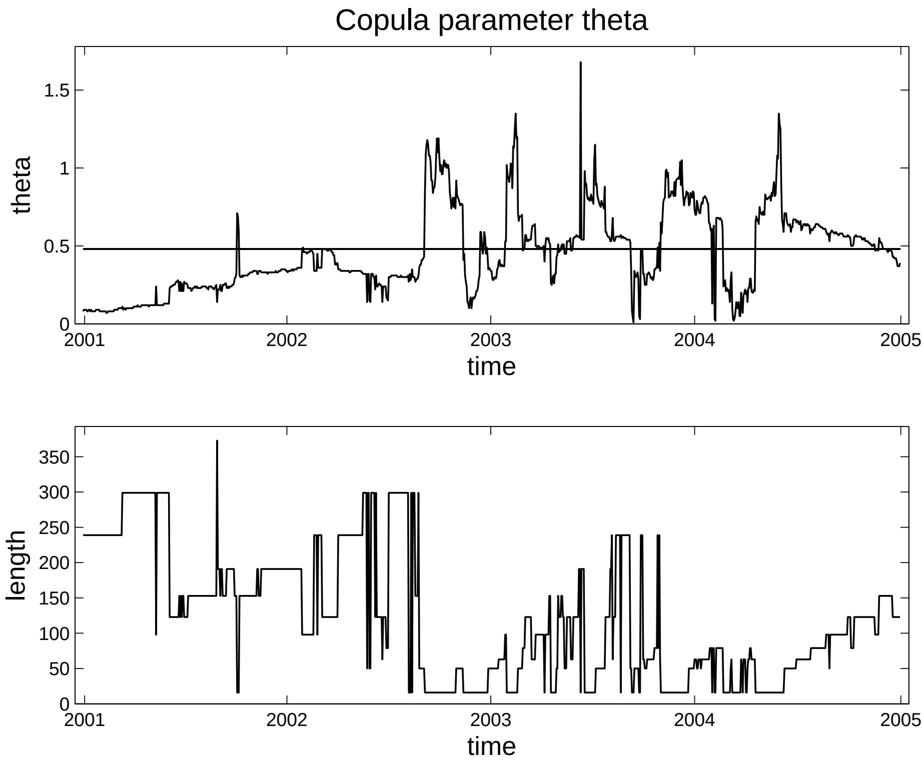


Figure 19: Upper panel: estimated copula dependence parameter θ for 6-dim data: Siemens, ThyssenKrupp, Schering, E.ON AG, Henkel, Lufthansa and the global parameter. Lower panel: estimated intervals of time homogeneity; with parameters $m_1 = 50$, $c = 1.25$ and $\alpha = 0.05$.

Portfolio	Exceedances ratio $\alpha(\times 10^2)$ 5.00	4.00	3.00	2.00	1.00
$(1,1,1,1,1,1)$	6.28	5.39	4.61	3.72	1.86
$(1,2,3,2,1,3)$	6.47	5.69	4.41	3.33	1.76
$(2,1,2,3,1,3)$	6.28	5.69	4.80	3.33	2.15
$(3,2,3,2,3,1)$	6.47	4.90	4.41	3.72	1.86
$(3,1,2,1,3,2)$	6.18	5.49	4.41	3.14	1.57
$(1,3,1,2,3,1)$	6.18	5.49	4.71	3.04	1.96
$(2,1,3,2,1,3)$	6.47	5.59	4.31	3.33	1.86
$(2,3,3,2,1,1)$	6.37	5.69	4.21	3.43	2.06
$(3,1,2,2,2,3)$	6.18	5.49	4.61	3.23	1.86
$(2,3,1,1,2,3)$	6.37	5.39	4.90	3.92	1.86
$(2,3,2,3,2,3)$	6.28	5.69	4.61	3.92	2.15
$(3,2,3,2,3,3)$	6.18	5.49	4.51	3.63	1.86
$(1,1,1,1,1,-1)$	5.88	5.39	4.21	3.72	1.57
$(1,2,3,2,1,-3)$	5.79	5.10	4.51	2.94	1.66
$(2,1,2,3,1,-3)$	6.37	5.20	4.31	3.23	1.76
$(3,2,3,2,3,-1)$	6.47	5.29	4.41	3.63	1.76
$(3,1,2,1,3,-2)$	6.08	5.10	4.41	3.14	1.66
$(1,3,1,2,3,-1)$	5.98	5.39	4.31	2.74	1.86
$(2,1,3,2,1,-3)$	5.88	5.29	4.61	3.04	1.76
$(2,3,3,2,1,-1)$	6.28	5.29	4.31	3.23	1.86
$(3,1,2,2,2,-3)$	5.69	4.90	4.31	3.72	1.76
$(2,3,1,1,2,-3)$	5.98	5.10	4.31	3.53	1.96
$(2,3,2,3,2,-3)$	5.88	5.49	4.12	3.82	1.76
$(3,2,3,2,3,-3)$	6.28	5.20	4.51	3.43	1.47
avg.	6.18	5.36	4.45	3.41	1.82
std.dev.	0.23	0.23	0.19	0.32	0.16
$\sum_{w\in W}(\hat{\alpha}-\alpha)^2$	0.34	0.46	0.51	0.50	0.16
$\sum_{w\in W}(\hat{\alpha}-\alpha)^2/\alpha$	6.96	11.55	17.26	25.34	16.93

Table 8: Exceedances ratio $\hat{\alpha}$ for different portfolios, estimated using *RiskMetrics* approach for 6-dim data: SIE, THY, SCH, EOA, HEN and LHA.

	Exceedances ratio $\alpha(\times 10^2)$				
Portfolio	5.00	4.00	3.00	2.00	1.00
$(1,1,1,1,1,1)$	5.98	5.39	4.61	3.72	1.66
$(1,2,3,2,1,3)$	5.88	5.10	3.63	2.35	1.37
$(2,1,2,3,1,3)$	7.26	6.47	5.49	3.92	2.25
$(3,2,3,2,3,1)$	5.59	5.00	4.31	3.33	1.66
$(3,1,2,1,3,2)$	6.08	5.20	4.31	3.33	1.47
$(1,3,1,2,3,1)$	5.10	4.02	3.14	2.84	1.57
$(2,1,3,2,1,3)$	6.96	5.59	4.51	3.33	2.06
$(2,3,3,2,1,1)$	6.57	5.29	4.61	3.63	1.96
$(3,1,2,2,2,3)$	7.16	6.18	5.39	4.41	2.15
$(2,3,1,1,2,3)$	7.06	5.98	5.10	3.82	1.96
$(2,3,2,3,2,3)$	6.86	5.59	4.71	3.33	2.06
$(3,2,3,2,3,3)$	5.98	5.20	4.41	3.43	1.57
$(1,1,1,1,1,-1)$	5.59	5.00	4.21	2.94	1.57
$(1,2,3,2,1,-3)$	4.71	4.12	3.23	2.15	1.37
$(2,1,2,3,1,-3)$	6.47	5.59	4.21	3.43	1.96
$(3,2,3,2,3,-1)$	5.69	4.80	4.21	3.14	1.66
$(3,1,2,1,3,-2)$	5.39	4.80	4.21	2.94	1.37
$(1,3,1,2,3,-1)$	4.31	3.53	2.94	2.45	1.66
$(2,1,3,2,1,-3)$	5.88	5.10	3.92	2.84	1.76
$(2,3,3,2,1,-1)$	6.18	5.29	4.41	3.23	1.96
$(3,1,2,2,2,-3)$	6.28	5.29	5.10	3.43	2.15
$(2,3,1,1,2,-3)$	5.69	5.00	4.31	2.74	1.66
$(2,3,2,3,2,-3)$	5.59	4.80	4.02	3.14	1.66
$(3,2,3,2,3,-3)$	5.59	4.90	3.63	2.64	1.66
avg.	5.99	5.13	4.28	3.19	1.76
std.dev.	0.75	0.64	0.65	0.52	0.26
$\sum_{w\in W}(\hat{\alpha}-\alpha)^2$	0.36	0.40	0.49	0.40	0.15
$\sum_{w\in W}(\hat{\alpha}-\alpha)^2/\alpha$	7.38	10.21	16.39	20.27	15.55

Table 9: Exceedances ratio $\hat{\alpha}$ for different portfolios, estimated with Clayton copula using moving window approach for 6-dim data: SIE, THY, SCH, EOA, HEN and LHA.

Portfolio	Exceedances ratio $\alpha(\times 10^2)$				
	5.00	4.00	3.00	2.00	1.00
$(1,1,1,1,1,1)$	5.49	4.61	3.82	2.84	1.37
$(1,2,3,2,1,3)$	5.10	3.92	3.23	2.55	1.37
$(2,1,2,3,1,3)$	6.77	5.49	4.61	3.23	2.25
$(3,2,3,2,3,1)$	5.20	4.21	3.43	2.45	1.37
$(3,1,2,1,3,2)$	5.29	4.61	3.43	2.25	1.37
$(1,3,1,2,3,1)$	4.02	3.23	2.64	2.45	1.17
$(2,1,3,2,1,3)$	5.98	5.10	4.12	2.94	1.57
$(2,3,3,2,1,1)$	5.88	5.00	4.12	3.23	1.66
$(3,1,2,2,2,3)$	6.77	5.49	4.71	3.14	1.66
$(2,3,1,1,2,3)$	6.57	5.49	4.41	2.94	1.37
$(2,3,2,3,2,3)$	6.37	4.71	3.82	2.94	1.57
$(3,2,3,2,3,3)$	5.29	4.31	3.72	2.74	1.37
$(1,1,1,1,1,-1)$	5.10	4.12	3.33	2.35	1.47
$(1,2,3,2,1,-3)$	4.51	3.82	2.74	2.06	1.37
$(2,1,2,3,1,-3)$	5.79	5.20	3.92	3.14	1.86
$(3,2,3,2,3,-1)$	5.10	4.21	3.23	2.45	1.37
$(3,1,2,1,3,-2)$	5.29	4.21	3.33	2.15	1.17
$(1,3,1,2,3,-1)$	3.72	2.94	2.55	2.15	1.37
$(2,1,3,2,1,-3)$	5.29	4.61	3.53	2.84	1.57
$(2,3,3,2,1,-1)$	5.29	4.90	4.02	3.23	1.57
$(3,1,2,2,2,-3)$	5.88	4.90	4.31	2.74	1.66
$(2,3,1,1,2,-3)$	5.39	4.80	3.63	2.45	1.57
$(2,3,2,3,2,-3)$	5.29	4.12	3.43	2.35	1.66
$(3,2,3,2,3,-3)$	5.00	4.12	2.94	2.15	1.37
avg.	5.43	4.51	3.63	2.66	1.50
std.dev	0.75	0.66	0.59	0.38	0.22
$\sum_{w\in W}(\hat{\alpha}-\alpha)^2$	0.17	0.16	0.17	0.13	0.07
$\sum_{w\in W}(\hat{\alpha}-\alpha)^2/\alpha$	3.53	4.08	5.85	6.97	7.42

Table 10: Exceedances ratio $\hat{\alpha}$ for different portfolios, estimated with Clayton copula using adaptive estimation procedure for 6-dim data: SIE, THY, SCH, EOA, HEN and LHA.

Method	Exceedances ratio $\alpha(\times 10^2)$				
	5.00	4.00	3.00	2.00	1.00
Riskmetrics	6.96	11.55	17.26	25.34	16.93
Moving Window	7.38	10.21	16.39	20.27	15.55
LCP	3.52	4.08	5.85	6.97	7.42

Table 11: Relative squared deviation $\sum_{w\in W}(\hat{\alpha}-\alpha)^2/\alpha$ for Riskmetrics, Moving Window and LCP approach (SIE, THY, SCH, EOA, HEN and LHA).

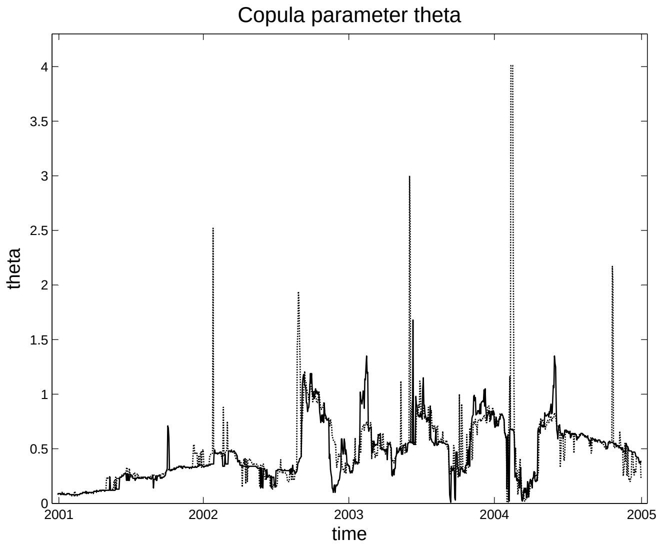


Figure 20: Estimated copula dependence parameter θ for 6-dim data: Siemens, ThyssenKrupp, Schering, E.ON AG, Henkel, Lufthansa estimated with parameter $m_1 = 20$ (dashed line) and $m_1 = 50$ (solid line).

However, we can observe higher standard deviations in the LCP case than in the moving window and Riskmetrics case.

We conclude by summarizing the main findings. The Clayton copula was used to estimate the Value-at-Risk from the 6-dimensional portfolio: at first, DCX, VW, ALV, MUV, BAY and BAS and then, SIE, THY, SCH, EOA, HEN and LHA with adaptive estimation and moving window approach. Backtesting was used to compare the performance of the copula-based Value-at-Risk estimation with the *RiskMetrics* approach. All three methods overestimate the Value-at-Risk in average. In terms of capital requirement, a financial institution would be requested to keep *more* capital aside than necessary to guarantee the desired confidence level. In the case of the portfolio consisting of DCX, VW, ALV, MUV, BAY and BAS, the Riskmetrics approach performed well, providing the relative squared deviation smaller than in the case of the moving window and the LCP procedure. However, one observes higher standard deviations in the case of Riskmetrics. For the second portfolio consisting of SIE, THY, SCH, EOA, HEN and LHA, Riskmetrics lead to

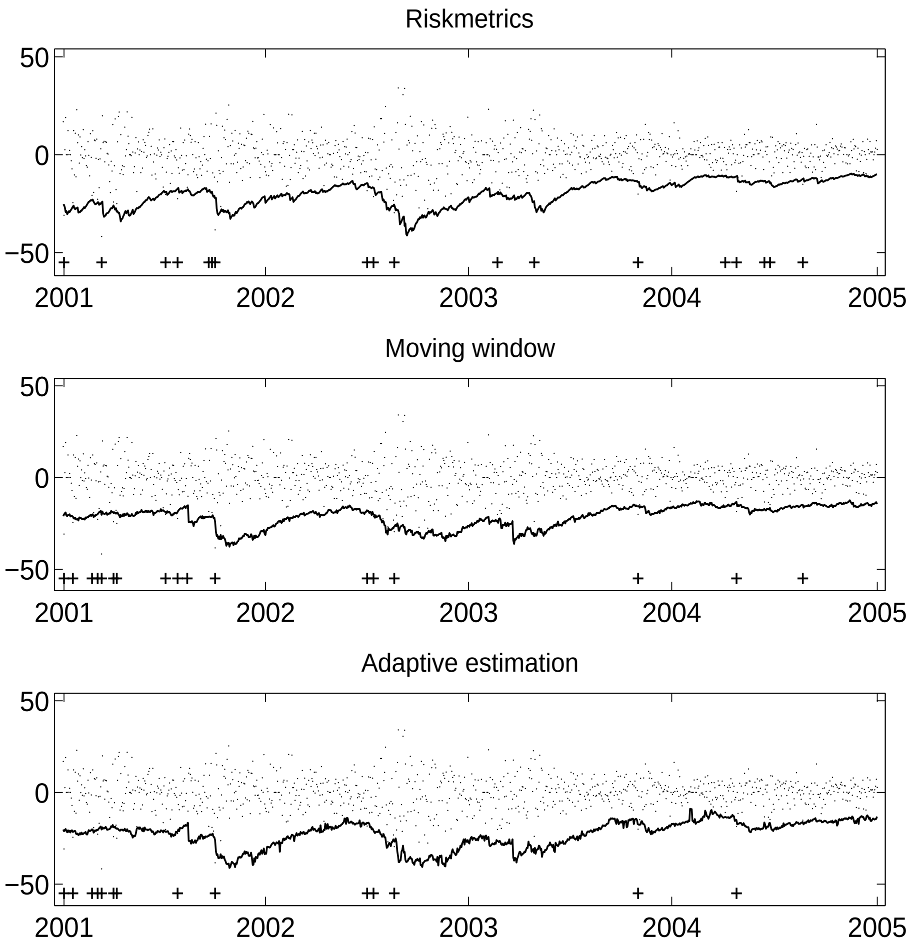


Figure 21: P&L (dots) and $\widehat{VaR}(\alpha)$ at level $\alpha_1 = 0.01$; $w = (3, 2, 3, 2, 3, -1)^\top$, estimated using *RiskMetrics* approach (upper panel), moving window approach (middle panel) and adaptive estimation procedure (lower panel) for 6-dim data: Siemens, ThyssenKrupp, Schering, E.ON AG, Henkel, Lufthansa.

smaller standard deviations fitting well only at the 5% level. It failed to capture the dependence at lower quantiles: the correlation structure contains nonlinearities that can not be captured by the multivariate normal distribution. Further, the adaptive estimation procedure allows for dynamic selection of the interval for dependence structure estimation and thus produces smaller relative squared deviations which leads to better backtesting results.

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